Acta Geodyn. Geomater., Vol. 3, No. 3 (143), 79-84, 2006

EARTH FREE OSCILLATIONS OBSERVED IN PLUMB LINE VARIATIONS FROM THE 26 DECEMBER 2004 EARTHQUAKE

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(*Received* January 2006, *accepted* March 2006)

ABSTRACT

The long water-tube tiltmeter observations of the plumb line variations were begun about four years ago. Since 2001, twice, the instrument registered strong plumb line variations associated with earthquake. Earthquake in September 2003 made opportunity for verification of hydrodynamic system of tiltmeter. We obtained in observations large number of cycle-slip effects, which made impossible correct interpretation of data during main phase of phenomenon. Results of tests of dumping system of water waving helped us to introduce some improvements in hydrodynamic system of instrument. Catastrophic earthquake with magnitude 9.1 which took place in the area of Indian Ocean on 26th December 2004 generated phenomenon of the Earth free oscillations associated with anomalous plumb line variations of the order of single milliseconds of arc (mas). The effects of surface waves passing associated with Earth free oscillations were registered by the tiltmeter as series of anomalous plumb line variations. Surface waves produced several milliseconds of arc amplitude of plumb line variations. Results of observations showed that low-pass filters considerably reduce contribution of short-period effects and effectively protect hydrodynamic system of instrument against resonance. Nevertheless during first two hours of phenomenon cycle-slip effects occurred several times. Plot of plumb line variations in space from 26th December 2004 showed us large complication of tilt signals as well as dominated role of tidal signal. Spectral analysis of the time series of the plumb line variations obtained with help of the long water-tube tiltmeter on 26 December 2004 appeared existing fundamental modes (n = 0) in the Earth free oscillations didn't exceed 0.2 mas.

KEYWORDS: geodynamic, earth tides, earth free oscillation, plumb line variations, long water-tube tiltmeter

1. INTRODUCTION

Several years of measurements carried out with the long water-tube tiltmeter confirm attractive features of this instrument (Kaczorowski 2004, 2005). The long water-tube construction provides us with measuring system whose internal accuracy of measurement is close to several thousandths of millisecond of arc (mas). Application of difference method for data processing eliminated the effect of instrumental drift. The comparison of results of observations from previous years confirmed the high stability of the measurement system sensitivity. We obtained a perfect correlation between tidal signals observed on 26 December 2004 and modeled tides calculated on the basis of observations from previous years (Figs. 4.2 and 4.3). Obtained discrepancies between plots are associated with non-tidal plumb line variations of the Earth free oscillations. Phenomenon of the Earth free oscillations is generated by extremely strong seismic events.

Dozen or so hours after earthquake MS equal 9.1 (Yearly – Catalogue of seismic data, 2004) on 26 December 2004 which took place near the coast of Sumatra - Andaman Islands we observed non-tidal plumb line variations. Earth free oscillations manifest by different phenomena such as body waves, surface waves, compressions and dilatations of the Earth medium (Pekeris and Jarosh, 1958; Alterman et al., 1959). Simultaneously, there are generated additional gravity and inertial accelerations as well as plumb line variations. Plumb line variations related to the Earth crust consist of tilt effects produced by surface waves and horizontal components of gravity and inertial accelerations.

2. PRINCIPLE OF OPERATION OF THE LONG WATER-TUBE TILTMETER

The principle of operation of the long water-tube tiltmeter takes advantage of fluids property (Moulton, 1919), concerning the response of fluid free surface under the influence of mass forces. The construction of the hydrodynamic system of tiltmeter assures that fluid, particularly water, preserves free surface along the full length of the system. We will present the behavior of the fluid with free surface under the influence of the field of potential forces resulting from the second-order tidal potential W_2. In the equilibrium state of fluid, the velocity $v = 0$ and the Euler equation of motion will take the form:

$$-\operatorname{grad} W_2 = \frac{1}{\rho} \operatorname{grad} p , \tag{2.1}$$

where p is the pressure, and ρ is the fluid density. Let us discuss the equation of state of a fluid, $\rho = \rho(p, t)$, where t is temperature. For isothermal or adiabatic processes, t = const or $t = t(p)$, we will get the equation of state in the form $\rho = \rho(p)$. For $\rho = \rho(p)$ there exists differentiable function $V(p)$ in the form of definite integral over the pressure variations $\int_{p_0}^{p} \frac{\mathrm{d}p}{\rho}$.

The gradient of function $V(p)$ is

$$\operatorname{grad} V(p) = \frac{1}{\rho} \operatorname{grad} p . \tag{2.2}$$

Substituting Eq. (2.2) into (2.1) we will get $\operatorname{grad}(V(p)) + \operatorname{grad}(W_2) = 0$. Hence,

$$V(p) + W_2 = \text{const} . \tag{2.3}$$

From Eq. (2.3) it follows that if the fluid surface is the equipotential surface W_2 = const, then the function $V(p)$ is constant. In such a case, from Eq. (2.2) it follows that p and $\rho(p)$ are constants. Thus, the fluid is in hydrostatic equilibrium state. Inversely, if the fluid is in hydrostatic equilibrium state, i.e., p = const and $\rho = \rho(p)$ = const, then by virtue of Eq. (2.2), function $V(p)$ is constant, so the tidal potential W_2 is also constant. Thus, in the hydrostatic equilibrium state, the free surface of the fluid is an equipotential surface. This law is the principle of operation of the long water-tube tiltmeter. When we measure the variations of the fluid free surface in the state close to hydrostatic equilibrium (quasi-equilibrium-approximation of adiabatic process), we simultaneously determine the variations of equipotential surface. The influence of tidal forces resulting from potential W_2 on the Earth globe causes, among other phenomena, the following two effects:

- Appearance of a horizontal component of gravity, which causes variation of the absolute direction of the plumb line;
- Deformation of the shape of the Earth and generation of absolute tilt of the Earth surface

We will apply solutions of the linear theory of elasticity for radially symmetric Earth model (Love, 1911). Changes of the absolute direction of the plumb line caused by tidal potential W_2 (Love, 1927), described with help of Love's numbers h and k, are as follows:

$$\alpha_{NS} = \frac{1+k}{g} \frac{1}{a} \frac{\partial W_2}{\partial \theta} \tag{2.4}$$

for the meridian component, and

$$\alpha_{EW} = \frac{1+k}{a\, g \sin\theta} \frac{\partial W_2}{\partial \lambda} \tag{2.5}$$

for the parallel component. At the same time, the absolute tilts of the Earth surface are:

$$\beta_{NS} = \frac{1}{g} \frac{h}{a} \frac{\partial W_2}{\partial \theta} \tag{2.6}$$

for the meridian component, and

$$\beta_{EW} = \frac{h}{a\, g \sin\theta} \frac{\partial W_2}{\partial \lambda} \tag{2.7}$$

for the parallel component. Considering both phenomena, the plumb line variations ψ measured in relation to the Earth's crust are:

$$\alpha_{NS} - \beta_{NS} = (1+k-h) \frac{1}{g} \frac{1}{a} \frac{\partial W_2}{\partial \theta} \tag{2.8}$$

for the meridian component, and

$$\alpha_{EW} - \beta_{EW} = (1+k-h) \frac{1}{g \sin\theta} \frac{1}{a} \frac{\partial W_2}{\partial \lambda} \tag{2.9}$$

for the parallel component (see Fig. 2.1).

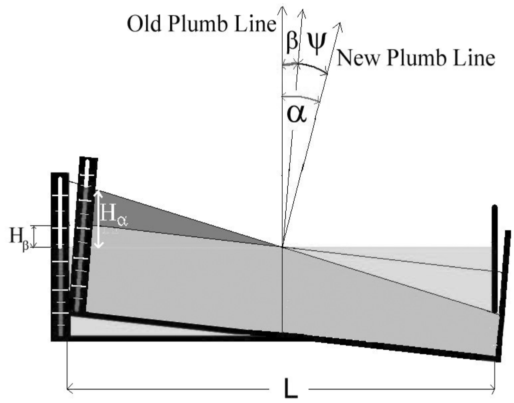

Fig. 2.1 Determination of angle ψ of plumb line variations on the basis of measurement of water level variations.

The combination of Love's numbers ($1+k-h$) in Eqs. (2.8) and (2.9) is the tidal amplitude factor γ of plumb line variations. Factors $(1/a)\,\partial W_2/\partial\theta$ and $(1/a\sin\theta)\,\partial W_2/\partial\lambda$ are horizontal components of the additional gravity, resulting from the second order tidal potential W_2. In the case of the long water-tube the variations of water free surface are measured relative to the reflected base lens. The lens is rigidly connected to the surrounding orogen as well as to the Earth's crust. The observed variations of water level H result from the two effects: changes H_α of water level caused by changes of absolute direction of the plumb line, and changes H_β of water level caused by tilts of the Earth surface (Fig. 2.1).

$$H = H_\alpha - H_\beta . \quad (2.10)$$

Dividing Eq. (2.10) by half length of the tube, $L/2$, we get the approximation for small angles:

$$\psi = \alpha - \beta \approx \frac{2}{L}\left(H_\alpha - H_\beta\right), \quad (2.11)$$

where ψ is the plumb line variation measured in relation to the Earth's crust. Therefore, on the basis of formula (2.11), measurements of the water level changes H allow us to determine angle $\psi = \alpha - \beta$ of plumb line variations. The quality of approximation of the water-surface to the equipotential surface depends on the rate of changes of the field of potential forces, the hydrodynamic parameters of the system, the viscosity of the applied fluid, as well as the level of local-origin disturbances. We made a large effort to adjust the hydrodynamic system of the long water-tube tiltmeter to measurements of the phenomena whose periods are longer than one minute.

3. INERTIAL EFFECTS PRODUCED BY SUMATRA - ANDAMAN EARTHQUAKE

The observations obtained during first two hours after seismic signal approach to instrument that is 01 hour and 11 minutes UT significantly differ with observations obtained in later phase of phenomenon. Over two hours after first seismic signals approach we observed strong inertial signals. Horizontal components of inertial signals produced tilts and rotation of the water surface in measurement chamber of instrument. Sequence of the following images showed us displacements of the centers of Newtonian rings as well as theirs deformations from circular into elliptical shape (Fig. 3.1). Duration of inertial impulses amounts single seconds only and did not affect water level in hydrodynamic system of tiltmeter. Mechanism of displacement of the Newtonian rings centre was shown in (Fig. 3.2).

Horizontal accelerations of the crust produce tilt of free surface of water in measurements chamber in

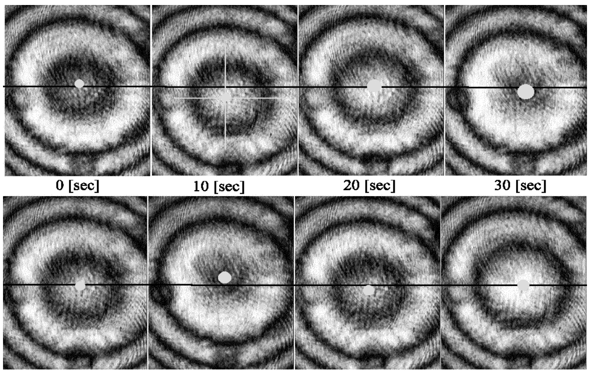

Fig. 3.1 Sequence of the following Newtonian rings obtained during first hour of Earth free oscillations on 26 December 2004 obtained from water-tube situated in azimuth $-121^0.4$.

relation to the optic system of interferometer. We are able to generate similar displacement effect by tilting interferometer in relation to the water surface. To obtain Newtonian rings displacement such as shown in Fig. 3.1 it is necessary to generate tilt of the order of single minute of arc. Therefore, the evaluate magnitudes of the observed inertial signals amount 0.3 gal. During later phase of free oscillations we did not observed inertial effects.

4. ANOMALOUS PLUMB LINE VARIATIONS ASSOCIATED WITH EFFECT OF THE EARTH FREE OSCILATIONS

Results of observations shown that low-pass filters inserted into hydrodynamic system considerably reduce contribution of short-period effects

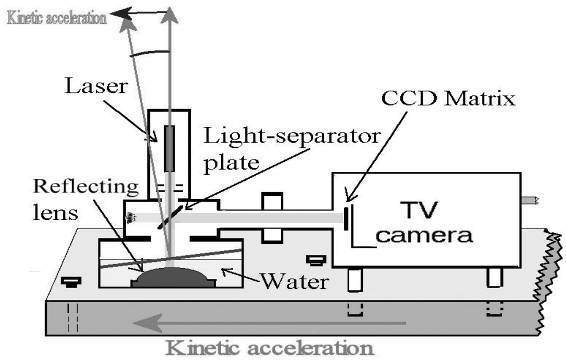

Fig. 3.2 Mechanism of generation of displacements of the Newtonian rings centre.

(smaller than single minute) and effectively protect hydrodynamic system against resonance. This circumstance enables us to register later phase of free oscillations (Figs. 4.1, 4.2, 4.3 and 5.1). During first two hours of phenomenon we observed rapid variations of water level caused by surface waves passing (periods of waves included between hundred seconds up to tens of minutes) as well as effects of additional gravity and inertial accelerations. Accidentally, variations of water level became too rapid to be firmly registered. Ten-second period of sampling interference picture was too long and we obtained significant number of cycle-slip errors. Two hours later cycle-slip errors became incidental.

In Figs. 4.2 and 4.3 there have been shown plots of plumb line variations registered by both tubes on 26 December 2004. Black color plots represent observed plumb line variations. Dark grey color plots describe noises. Light grey color present modeled tide calculated on the basis of ephemerid obtained on the basis of previous year observations. Results of subtraction tidal signals from observations were shown in Figs. 4.4. and 4.5. Upper plots shown non-tidal origin plumb line variations associated with free oscillations. Maximum observed amplitude reached 5 mas and decreased to 0.5 mas after two and a half

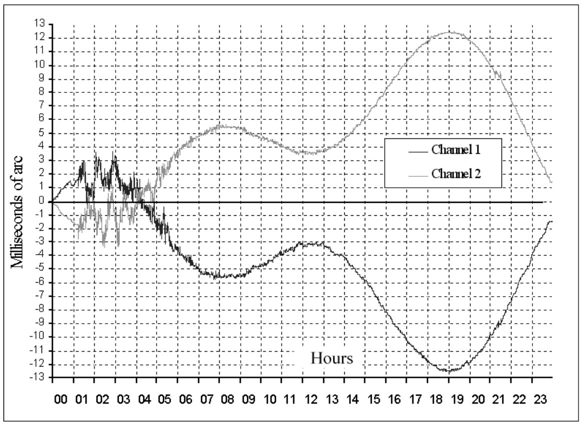

Fig. 4.1 Plots of water level variations recalculated on milliseconds of arc of plumb line variations obtained from two ends of the tube (channels 1 and 2) placed in azimuth –121^0.4.

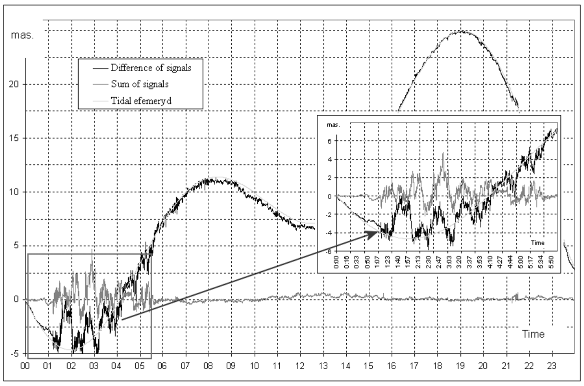

Fig. 4.2 The plumb line variations in azimuth -121^0.4 in 26 December 2004. Visible anomaly of plumb line variations associated with Earth free oscillations.

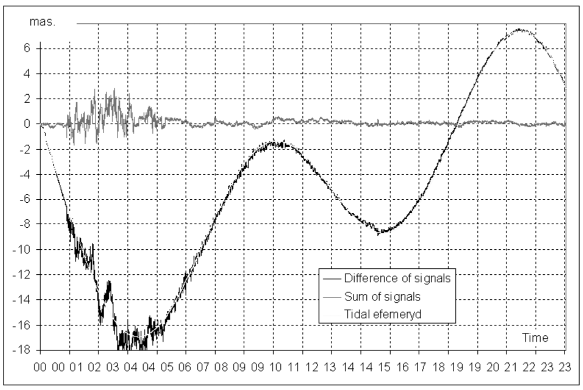

Fig. 4.3 The plumb line variations in azimuth –31^0.4 in 26 December 2004. Visible anomaly of plumb line variations associated with Earth free oscillations.

hours. Lower plots in Figs. 4.4. and 4.5 describe level of noises. These plots contain errors of data elaborations caused by cycle-slip effects mainly. During first two hours amplitudes of errors caused by cycle-slip effects were close to amplitudes of plumb line variations. In later phase of phenomenon the level of noises decreases to tenth of mas.

On the basis of data obtained from both water-tubes we assembled plot of plumb line variations in space (Fig. 5.1). On this figure the tubes are situated exactly in axis of coordinates. Plumb line variations were shown in azimuths of measurements –azimuths of the tubes –31.4 deg and –121.4 deg. Magnified fragment of plot shows large complications of tilts signals affecting the instrument. Presented fragment was chosen from end of third hour of phenomenon and spread over fifteen minutes. During this interval amplitude of non-tidal plumb line variations was close to single mas. Then, amplitudes of non-tidal tilt waves reduced to several tenth of mas.

From Fig.5.1 we can notice that non-tidal signals slightly affected tidal trends and that tilt waves associated with free oscillations cause only around-tidal variations of plumb line. The mathematical foundations of solutions of the spherically symmetrical bodies were given by Love (1927). Later authors (Pekeris et al., 1958 and Alterman et al., 1959) applied Love's theory to more realistic self-gravitating models of the Earth. Approximation of the Earth by the spherically symmetrical model allowed us to express free oscillations in terms of spherical harmonics. For spherical model free oscillations take forms spheroidal $_n\mathbf{S}_{lm}$ and Toroidal $_n\mathbf{T}_{lm}$ vibrations. Coefficients: n is the radial overtone number, l is the angular degree and m is the azimuthal order number. Spectral analysis of the time series of the plumb line variations obtained with help of the long water-tube tiltmeter on 26 December 2004 appeared existing fundamental modes (n=0) in the Earth free oscillations. Amplitudes of these modes varied in the time and didn't exceed 0.2 mas (Fig. 5.2). On the figure spheroidal $_0\mathbf{S}_2$ and $_0\mathbf{S}_3$ and Toroidal $_0\mathbf{T}_2$ vibrations of free oscillations were shown.

CONCLUSIONS

Several years of measurements carry out with help of the long water-tube tiltmeter confirm attractive features of this instrument (Kaczorowski, 2004, 2005). Effort put into construction of the long water-tube provides us with measuring system which internal accuracy of measurements is close to several thousandths of mas. Application of difference method for data processing caused elimination effect of instrumental drift. The comparison of results of observations from previous years confirmed high stability of sensitivity of measure system. We can confirm perfect correlation between tidal signals observed on 26 December 2004 and modeled tides calculated on the basis of observations from previous years (Figs. 4.2 and 4.3). Discrepancies arrived between

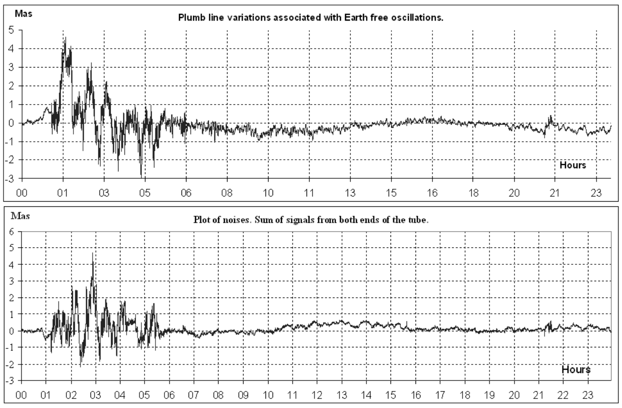

Fig. 4.4 Plots of differences and sum of signals from both ends of the tube in azimuth -121^0.4 after subtraction tidal signal. Differences of signals (upper plot) represent effect of plumb line variations caused by Earth free oscillations. Sum of signals (lower plot) represents level of noises.

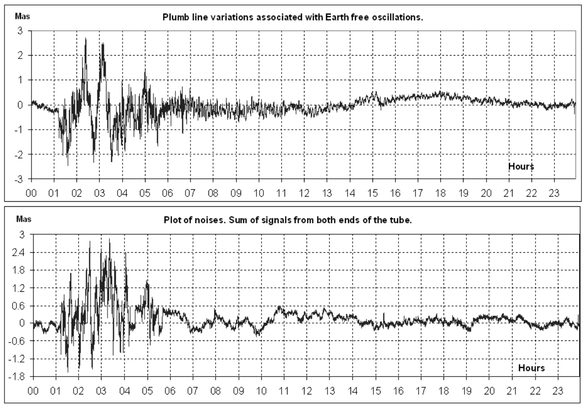

Fig. 4.5 Plots of differences and sum of signals from both ends of the tube in azimuth -31^0.4 after subtraction tidal signal. Differences of signals (upper plot) represent effect of plumb line variations caused by Earth free oscillations. Sum of signals (lower plot) represents level of noises.

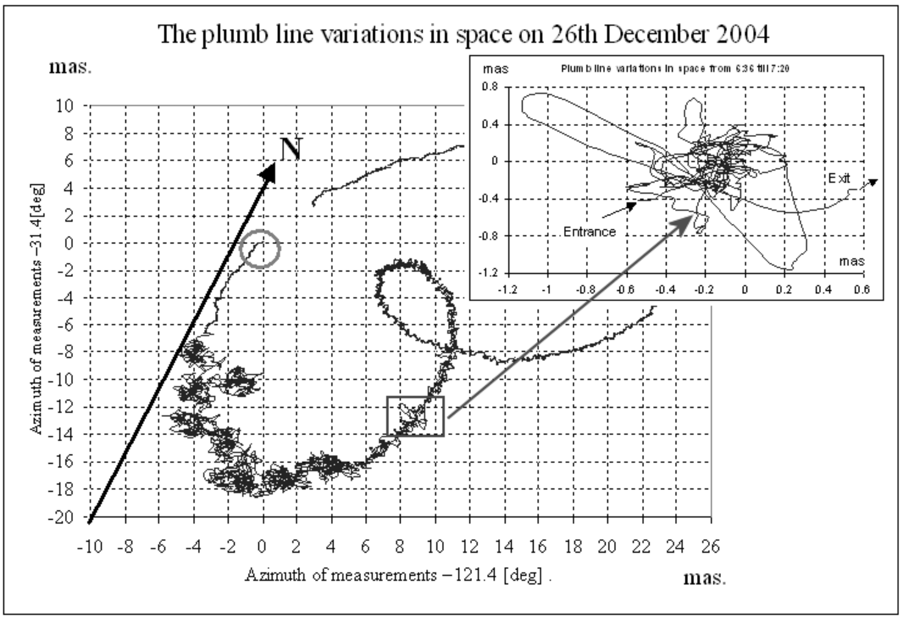

Fig. 5.1 Plot of plumb line variations in space assembled on the basis of measurements from both water-tubes.

plots are associated with non-tidal plumb line variations caused by the Earth free oscillations. Several years long experience provided us information about positive features of new instrument as well as information about some drawbacks of measure system. There are two main problems of measurements: the cycle-slip effects as well as arising pressure gradient along the water-tubes of instrument. Pressure gradient disturbs water free surface through the inverse barometric effect. Gradient of the order of 10^{-6} Hpa/meter can be erroneously interpreted as plumb line variations. Both the problems are possible to solve. Cycle-slip effects will by eliminated by means of change of the method of images registration from regular (every ten second) into irregular control by phases variations between the following interference images. Application of a new system of registration causes reduction number of cycle-slip effects as well as diminishes the number of storing images. The second problem related to pressure gradient in underground tunnels can be solved by means of

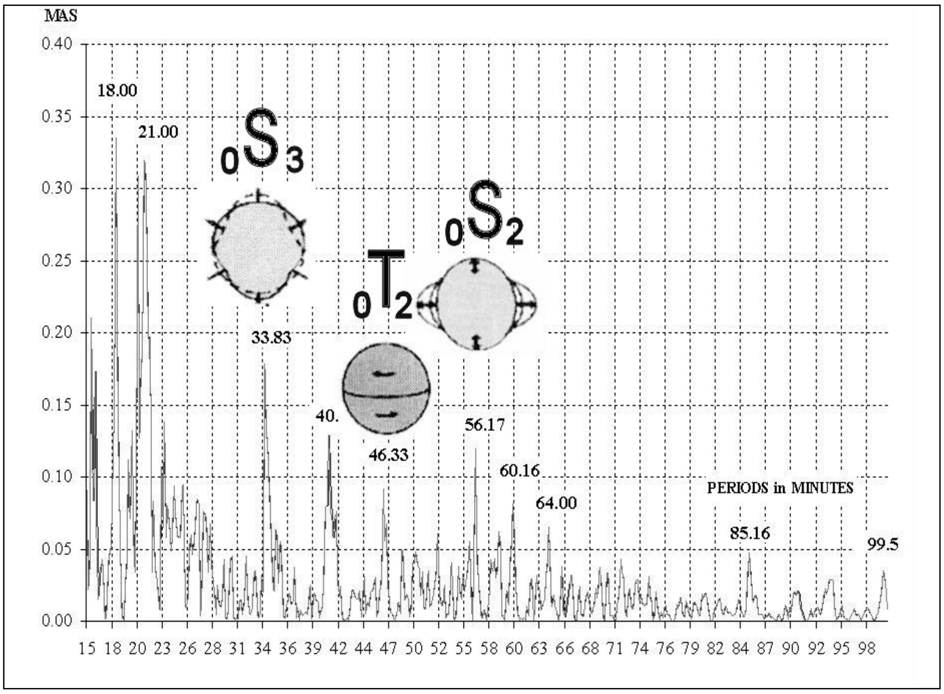

Fig. 5.2 Power Spectrum of plumb line variations on 26 December 2004 in azimuth - $121^0.4$. There have been shown spheroidal $_0\mathbf{S}_2$ and $_0\mathbf{S}_3$ and Toroidal $_0\mathbf{T}_2$ fundamental modes (n=0) of free oscillations. Small icons describe shape of deformations of the Earth body associated with spherical harmonics.

limitation of air exchange between underground and outside. The phenomenon of air pressure compensate-ion in the underground plays main role in generating of the water free surface disturbances producing signals of 0.5 mas order (values of mean square errors). For this purpose we are going to build two partition walls separating tunnels with instrument from remind part of underground. Obtained by us experiences with the long water-tube tiltmeter confirm particular properties of new instrument such as high sensitivity, lack of instrumental drift, constant and well determined sensitivity of measuring system, constant and well known azimuth of measurements as well as effectiveness of dumping system for short periods seismic effects. Simultaneously, differential method of data elaboration helps us to appreciate the level of errors of instrumental origin. Mentioned particularities of the long water-tube confirmed predestination of our instrument for investigation wide range of the long period geodynamic phenomena such as Earth free oscillations, tidal effects, as non-periodic loading effects of atmospheric origin as well as tectonic plates motions.

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